Online Supplemental Material to "Why is Trade Not Free? A Revealed Preference Approach"

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This appendix outlines our algorithm to solve the equilibrium given a set of trade taxes and exogenous parameters (Section E), and presents the expressions used to compute the sensitivity of real income to imports (Section F).

E Numerical Algorithm for Equilibrium Computation

This section describes the algorithm that we use to compute equilibria with zero tariffs in Section 4.3, in order to construct instrumental variables, as well as equilibria without redistributive trade protection in Section 5. Given the multiplicative structure of the model, it is convenient to work with ad-valorem tariffs $\{t_{ih}^{\text{av}}\}$. Formally, the equilibrium conditions are those described in Appendix C, except for the two equations (C.11) and (C.17) in which specific tariffs enter explicitly, which we write as

$$\tau = \frac{1}{N} \sum_{i \in \mathcal{R}_F} \sum_{r \in \mathcal{R}_H} \sum_{s \in \mathcal{S}} \sum_{h \in \mathcal{H}_s} \frac{t_{ih}^{\text{av}}}{1 + t_{ih}^{\text{av}}} X_{irsh}^F, \tag{E.1}$$

$$(p_{irh})^{1+\psi^{X,F}} = \frac{t_{ih}^{\text{av}}}{1 + t_{ih}^{\text{av}}} (p_{irh})^{1+\psi^{X,F}} + \theta_{irh}^{X,F} (X_{irh}^F)^{\psi^{X,F}}. \tag{E.2}$$

The algorithm solves for the equilibrium conditional on arbitrary ad-valorem tariffs $\{\tilde{t}_{ih}^{\text{av}}\}$ as well as the calibrated values of $\{\alpha_s, \alpha_{ks}, \gamma_s, \xi_{rs}, \theta_{rds}, \theta_{orsh}^c, \theta_{irh}^{X,F}, \theta_{rih}^{M,F}, L_{rs} \equiv \phi_{rs} N_{rs}, N_{rs}\}$ and $\{\sigma, \psi^{X,F}, \psi^{M,F}\}$.[2] For the counterfactual with zero tariffs, we set $\tilde{t}_{ih}^{\text{av}} = 0$. For the counterfactual without redistributive trade protection, we set $\tilde{t}_{ih}^{\text{av}} = t'_{ih}$, where t'_{ih} is described in equation (21).[3]

E.1 Home and Foreign Price Indices and Expenditures

In what follows, it will be convenient to work with sub-price indices for the Home and Foreign varieties within each sector purchased by each region. Concretely, equations (C.1)-(C.3)

[2] In what follows, we work with L_{rs} instead of ϕ_{rs} as a way to handle cases in which $N_{rs} = 0$ but $\phi_{rs} N_{rs} > 0$.

[3] Since we have normalized import prices to one in the initial equilibrium and set $\psi^{X,F} = 0$, we note that the world prices of Home's imports in the counterfactual equilibrium without redistribution still satisfy $p_{irh}^{X,F} \equiv 1$. So specific and ad-valorem tariffs are equivalent in the counterfactual equilibrium.

imply

$$p_{rs} = [\alpha_s]^{-\alpha_s}[w_{rs}]^{\alpha_s}\prod_{k\in S}[\alpha_{ks}]^{-\alpha_{ks}}[P_{rk}]^{\alpha_{ks}}, \tag{E.3}$$

$$P_{rk} = \left[\sum_{c=H,F}\theta^c_{rk}[P^c_{rk}]^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{E.4}$$

$$P^H_{rk} = \left[\sum_{o\in\mathcal{R}_H}\tilde{\theta}^H_{ork}[p_{ok}]^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{E.5}$$

$$P^F_{rk} = \left[\sum_{v\in\mathcal{H}_k}\sum_{o\in\mathcal{R}_F}\tilde{\theta}^F_{orkv}[p_{orv}]^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{E.6}$$

where $\theta^c_{rk} \equiv \sum_{v\in\mathcal{H}_k}\sum_{o\in\mathcal{R}_c}\theta^c_{orkv}$, $\tilde{\theta}^H_{ork} \equiv \sum_{v\in\mathcal{H}_k}\theta^H_{orkv}/\theta^H_{rk}/(\theta_{orv})^{1-\sigma}$, and $\tilde{\theta}^F_{orkv} \equiv \theta^F_{orkv}/\theta^F_{rk}$. These definitions and equations (C.6)-(C.7) allow us to express expenditures as

$$X^F_{irsh} = \frac{\tilde{\theta}^F_{irsh}[p_{irh}]^{1-\sigma}}{[P^F_{rs}]^{1-\sigma}}X^F_{rs} \quad \text{and} \quad X^F_{rs} = \frac{\theta^F_{rs}[P^F_{rs}]^{1-\sigma}}{[P_{rs}]^{1-\sigma}}X_{rs}, \tag{E.7}$$

$$X^H_{ors} = \frac{\tilde{\theta}^H_{ors}[p_{os}]^{1-\sigma}}{[P^H_{rs}]^{1-\sigma}}X^H_{rs} \quad \text{and} \quad X^H_{rs} = \frac{\theta^H_{rs}[P^H_{rs}]^{1-\sigma}}{[P_{rs}]^{1-\sigma}}X_{rs}. \tag{E.8}$$

E.2 Solving for Domestic Demand

A key step in characterizing equilibrium outcomes is to solve for the domestic spending $\{X_{rs}\}$ as a function of $\{p_{rs}\}$ and $\{P^F_{rs}\}$. This is a fixed point problem because spending (non-linearly) affects tariff revenue, which in turn affects spending.

We begin by characterizing tariff revenue as a function of p_{rs} and P^F_{rs} and X_{rs}. From the expressions above and in Appendix C, we get that

$$X^F_{irsh} = \frac{\tilde{\theta}^F_{irsh}[p_{irh}]^{1-\sigma}}{[P^F_{rs}]^{1-\sigma}}X^F_{rs},$$

$$p_{irh} = \left[\theta^{X,F}_{irh}(1+\tilde{t}^{\text{av}}_{ih})\right]^{\frac{1}{1+\psi^{X,F}}}(X^F_{irsh})^{\frac{\psi^{X,F}}{1+\psi^{X,F}}}. \tag{E.9}$$

Thus,

$$p_{irs} = \left[\theta^{X,F}_{irh}(1+\tilde{t}^{\text{av}}_{ih})\right]^{\frac{1}{1+\psi^{X,F}\sigma}}\left(\tilde{\theta}^F_{irsh}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}}\left(\frac{X^F_{rs}}{[P^F_{rs}]^{1-\sigma}}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}},$$

$$X^F_{irsh} = \left[\theta^{X,F}_{irh}(1+\tilde{t}^{\text{av}}_{ih})\right]^{\frac{1-\sigma}{1+\psi^{X,F}\sigma}} \left(\tilde{\theta}^F_{irsh}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}} \left(\frac{X^F_{rs}}{[P^F_{rs}]^{1-\sigma}}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}.$$

Let us write

$$X^F_{irsh} = \varphi_{irsh} \left(\frac{X^F_{rs}}{[P^F_{rs}]^{1-\sigma}}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}, \tag{E.10}$$

$$\text{where} \quad \varphi_{irsh} \equiv \left[\theta^{X,F}_{irh}(1+\tilde{t}^{\text{av}}_{ih})\right]^{\frac{1-\sigma}{1+\psi^{X,F}\sigma}} \left(\tilde{\theta}^F_{irsh}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}. \tag{E.11}$$

Since $\left(P^F_{rs}\right)^{1-\sigma} = \sum_{h\in\mathcal{H}_s}\sum_{i\in\mathcal{R}_F} \tilde{\theta}^F_{irsh}[p_{irh}]^{1-\sigma}$,

$$P^F_{rs} = \left(X^F_{rs}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}}} \left[\sum_{h\in\mathcal{H}_s}\sum_{i\in\mathcal{R}_F} \varphi_{irsh}\right]^{\frac{1+\psi^{X,F}\sigma}{(1+\psi^{X,F})(1-\sigma)}}. \tag{E.12}$$

Thus,

$$X^F_{irsh} = \frac{\varphi_{irsh}}{\sum_{v\in\mathcal{H}_s}\sum_{o\in\mathcal{R}_F}\varphi_{orsv}} X^F_{rs}. \tag{E.13}$$

Finally, $X^F_{rs} = \frac{\theta^F_{rs}[P^F_{rs}]^{1-\sigma}}{[P_{rs}]^{1-\sigma}} X_{rs}$ implies that

$$X^F_{rs} = \left(\theta^F_{rs}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}} \mu_{rs} \left(\frac{X_{rs}}{[P_{rs}]^{1-\sigma}}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}, \tag{E.14}$$

$$\text{where} \quad \mu_{rs} \equiv \sum_{h\in\mathcal{H}_s}\sum_{i\in\mathcal{R}_F} \varphi_{irsh}. \tag{E.15}$$

Thus, substituting (E.14) into (E.12), we obtain

$$P^F_{rs} = \zeta_{rs} \left(\frac{X_{rs}}{[P_{rs}]^{1-\sigma}}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}}, \tag{E.16}$$

$$\text{where} \quad \zeta_{rs} \equiv \left(\theta^F_{rs}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}} [\mu_{rs}]^{\frac{1}{1-\sigma}}. \tag{E.17}$$

Combining (E.13) and (E.14) we can also write

$$X^F_{irsh} = \varphi_{irsh} \left(\theta^F_{rs}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}} \left(\frac{X_{rs}}{[P_{rs}]^{1-\sigma}}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}, \tag{E.18}$$

and, using $T_{rs} \equiv \sum_{h \in \mathcal{H}_s} \sum_{i \in \mathcal{R}_F} \frac{\tilde{t}^{\text{av}}_{ih}}{1+\tilde{t}^{\text{av}}_{ih}} X^F_{irsh}$,

$$T_{rs} = \left(\frac{X_{rs}}{[P_{rs}]^{1-\sigma}} \right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}} \varphi^R_{rs}, \tag{E.19}$$

with

$$\varphi^R_{rs} \equiv \left(\theta^F_{rs} \right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}} \sum_{h \in \mathcal{H}_s} \sum_{i \in \mathcal{R}_F} \frac{\tilde{t}^{\text{av}}_{ih}}{1+\tilde{t}^{\text{av}}_{ih}} \varphi_{irsh}. \tag{E.20}$$

Combining (C.13) and (E.19), we can solve for domestic expenditure $\{X_{rs}\}$ as the solution to:

$$X_{rs} - \sum_{d \in \mathcal{R}_H} \sum_{k \in \mathcal{S}} \hat{e}_{rs,dk} (X_{dk})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\kappa}} = \hat{X}_{rs}, \tag{E.21}$$

with

$$\begin{aligned} \hat{X}_{rs} &= \xi_{rs} + \sum_{k \in \mathcal{S}} (\gamma_s \alpha_k + \alpha_{sk}) Y_{rk}, \\ \hat{e}_{rs,dk} &= \gamma_s \frac{N_r}{N} \varphi^R_{dk} \left([P_{dk}]^{\sigma-1} \right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}, \end{aligned} \tag{E.22}$$

where both $\{Y_{rk}\}$ and $\{P_{dk}\}$ are only functions of $\{p_{rs}\}$ and $\{P^F_{rs}\}$ via (C.9) and (E.4)–(E.6).

E.3 Algorithm

We use the following algorithm to solve for the competitive equilibrium of the model.

i. Compute parameters that are invariant to prices: φ^R_{rs} from (E.20), ζ_{rs} from (E.17), μ_{rs} from (E.15), φ_{irsh} from (E.11), and δ_{rs} from (C.15).

ii. We have an outer loop (indexed by a). Guess $P^{F,a=0}_{rs}$ using (E.16):

$$P^{F,a=0}_{rs} = \zeta_{rs} \left(\tilde{E}^0_{rs} \right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}}, \tag{E.23}$$

where we use a pre-determined choice of the sector-level demand shifter $\tilde{E}^0_{rs} \equiv X^0_{rs} (P^0_{rs})^{\sigma-1}$ (which we take to be the value in some observed initial equilibrium).

iii. Given $P^{F,a}_{rs}$, we have a middle loop (indexed by b) that solves for p^a_{rs}.

(a) We guess $p_{rs}^{a,b=0} = \theta_{rds}$ for arbitrary d.

(b) Given $\{P_{rs}^{F,a}, p_{rs}^{a,b}\}$, compute the following vectors of region-sector variables (with length $|\mathcal{R}_H| \cdot |\mathcal{S}|$ and same ordering of sectors and regions for all variables).

  i. Domestic region-sector price index $P_{rs}^{H,a,b}$ by substituting $p_{rs}^{a,b}$ into (E.5).

  ii. Region-sector price index $P_{rs}^{a,b}$ by substituting $P_{rs}^{F,a}$ and $P_{rs}^{H,a,b}$ into (E.4).

  iii. Region-sector supply $Y_{rs}^{a,b}$ by substituting $p_{rs}^{a,b}$ and $P_{rk}^{a,b}$ into (C.9).

  iv. Region-sector foreign demand $D_{rs}^{F,a,b}$ by substituting $p_{rs}^{a,b}$ into (C.15).

  v. Region-sector spending $X_{rs}^{a,b}$ using (E.21). Here, we have an inner fixed-problem algorithm (indexed by c):

    A. Compute $\hat{X}^{a,b} = [\hat{X}_{rs}]_{|\mathcal{R}_F|\cdot|\mathcal{S}|\times 1}$ and $\hat{e}^{a,b} \equiv [\hat{e}_{rs,dk}^{a,b}]_{|\mathcal{R}_F|\cdot|\mathcal{S}|\times|\mathcal{R}_F|\cdot|\mathcal{S}|}$ by plugging $Y_{rk}^{a,b}$ and $P_{dk}^{a,b}$ into (E.22).

    B. Guess that $X^{a,b,c=0} = \sum_{d=0}^{\bar{d}} (\hat{e}^{a,b})^d \hat{X}^{a,b}$.[4] Given $X^{a,b,c}$, compute

$$\widetilde{X}_{rs}^{a,b,c} \equiv \sum_{d\in\mathcal{R}_H} \sum_{k\in\mathcal{S}} \hat{e}_{rs,dk}^{a,b} \left(X_{d,k}^{a,b,c}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}} + \hat{X}_{r,s}^{a,b},$$

$$Xerr_{rs} \equiv X_{rs}^{a,b,c} - \widetilde{X}_{rs}^{a,b,c}.$$

    If $max_{r,s}|Xerr_{rs}| < tol$, then we set $X^{a,b} = X^{a,b,c}$. Otherwise, we repeat the step with

$$X_{rs}^{a,b,c+1} = X_{rs}^{a,b,c} - \chi^X \left(X_{rs}^{a,b,c} - \widetilde{X}_{rs}^{a,b,c}\right).$$

    for $\chi^X > 0$ small enough. Note that, given our initial guess of $X^{a,b,c=0}$, this converges in a single step if $\psi^{X,F} = 0$.

  vi. Region-sector domestic spending $X_{rs}^{H,a,b}$ by substituting $P_{rs}^{H,a,b}$, $P_{rs}^{a,b}$ and $X_{rs}^{a,b}$ into (E.8).

  vii. Region-sector domestic demand $D_{rs}^{H,a,b}$ by substituting $p_{rs}^{a,b}$, $X_{rs}^{H,a,b}$ and $P_{rs}^{H,a,b}$ into (E.8) and (C.14).

  viii. Region-sector excess supply:

$$Yerr_{rs}^{a,b} \equiv \frac{Y_{rs}^{a,b} - (D_{r,s}^{F,a,b} + D_{rs}^{H,a,b})}{Y_{rs}^{a,b}}.$$

(c) If $\max_{r,s}\{|Yerr_{rs}^{a,b}|\} < tol$, then proceed to step (iv) by setting $p_{rs}^{a} = p_{rs}^{a,b}$. If not,

[4] $\bar{d}$ is a numerical parameter that can in principle be set to infinity. In practice, we set $\bar{d} = 5$.

repeat (iii.b) with the updated guesses for prices:

$$\ln p_{rs}^{a,b+1} = \ln p_{rs}^{a,b} - \chi^H Yerr_{rs}^{a,b}$$

for $\chi^H > 0$ small enough. Intuitively, supply is larger than demand when $Yerr_{rs}^{a,b} > 0$, so we reduce domestic prices in region r sector s until convergence.

iv. Given the demand shifter $E_{rs}^a = X_{rs}^a (P_{rs}^a)^{\sigma-1}$, we compute the error in the import price index of each region-sector,

$$Perr_{rs} \equiv |P_{rs}^{F,a} - \zeta_{rs} \left(E_{rs}^a\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}} |.$$

If $\max_{r,s}\{|Perr_{rs}|\} < tol$, then stop. If not, repeat step (iii) with the updated guess for prices:

$$P_{rs}^{F,a+1} = \zeta_{rs} \left(\tilde{E}_{rs}^{a+1}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}},$$

with

$$\tilde{E}_{rs}^{a+1} = \tilde{E}_{rs}^a - \chi^F (\tilde{E}_{rs}^a - E_{rs}^a)$$

for $\chi^F > 0$ small enough.

v. Upon convergence, we compute X_{irsh}^F using (E.10) and (E.14), import prices p_{irh} using (E.9), import quantity $m_{irh} = X_{irsh}^F / p_{irh}$, region-sector value-added W_{rs} and wages $w_{rs} = \frac{W_{rs}}{L_{rs}}$ (with $L_{rs} = \phi_{rs} N_{rs}$) using (C.8), region-level consumption price index P_r^C using (C.12), and per-capita lump-sum transfers $\tau = \frac{1}{N} \sum_{r \in \mathcal{R}_H, s \in \mathcal{S}} T_{rs}$ with T_{rs} given by (E.19).

F Analytical Jacobian Matrices

F.1 Jacobians with Respect to Imports

In this section, we derive analytical expressions for the derivatives of imports, wages, consumption price indices, country-specific terms-of-trade, and fiscal externalities with respect to imports of each country-product variety. Our derivation is based on the linearization of three key blocks of the model: an import pricing block, a domestic output market clearing block, and a domestic expenditures block. Linearizing these three blocks allows us to compute the Jacobians of P_{rs}^F, p_{rs}, and X_{rs} with respect to tariffs $1 + t_{ih}^{\text{av}}$. More straightforward linearizations then connect changes in these variables to changes in our variables of interest as well as—critically—changes in imports. We then invert the relationship between imports and tariffs to solve for the Jacobians of our variables of interest with respect to imports.

F.1.1 Jacobians with Respect to Tariffs

We now derive the Jacobians of P^F_{rs}, p_{rs}, and X_{rs} with respect to tariffs $1 + t^{\text{av}}_{ih}$. We do so by first linearizing three key blocks of the model and then combining these blocks.

Import Pricing Block. The first equation relates changes in spending, changes region-sector prices, changes in Foreign price indices, and changes in tariffs using equations (E.16), (E.17), (E.15), (E.11), (E.4) and (E.5). We have

$$P^F_{rs} = \zeta_{rs} \left(\frac{X_{rs}}{[P_{rs}]^{1-\sigma}} \right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}}$$

$$\text{where} \quad \zeta_{rs} \equiv \left(\theta^F_{rs}\right)^{\frac{\psi^{X,F}}{1+\psi^{X,F}\sigma}} (\mu_{rs})^{\frac{1}{1-\sigma}}$$

$$\mu_{rs} \equiv \sum_{h \in \mathcal{H}_s} \sum_{i \in \mathcal{R}_F} \varphi_{irsh}$$

$$\varphi_{irsh} \equiv \left[\theta^{X,F}_{irh}(1 + \tilde{t}^{\text{av}}_{ih})\right]^{\frac{1-\sigma}{1+\psi^{X,F}\sigma}} \left(\tilde{\theta}^F_{irsh}\right)^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}$$

$$P_{rk} = \left[\sum_{c=H,F} \theta^c_{rk} [P^c_{rk}]^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

$$P^H_{rk} = \left[\sum_{o \in \mathcal{R}_H} \tilde{\theta}^H_{ork} [p_{ok}]^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

To linearize this system of equations, first define the $|\mathcal{R}_F||\mathcal{S}|$-dimensional matrices and vectors

$$[\boldsymbol{\Omega^H}]_{rs,ok} \equiv \frac{X^H_{ors}}{X^H_{rs}} \mathbb{1}_{k=s} \qquad [\boldsymbol{s^F}]_{rs} \equiv \frac{X^F_{rs}}{X_{rs}}$$

$$[\boldsymbol{\Omega^F}]_{rs,ih} \equiv \frac{X^F_{irsh}}{X^F_{rs}} \qquad [\tilde{\boldsymbol{t}}]_{ih} \equiv \tilde{t}^{\text{av}}_{ih}$$

where $\text{Diag}(\boldsymbol{x})$ denotes the diagonal matrix whose entries are given by the vector $\boldsymbol{x}$. Log-linearizing these equations implies the following matrix equation:

$$
\begin{aligned}
\boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{P^F}} d\log \boldsymbol{P}^F &= \boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{X}} d\log \boldsymbol{X} + \boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{p}} d\log \boldsymbol{p} + \boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{1+t^{av}}} d\log(\boldsymbol{1}+\tilde{\boldsymbol{t}}) \qquad \text{(F.1)}\\
\text{where} \quad \boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{P^F}} &\equiv \left((1+\psi^{X,F}\sigma)\boldsymbol{I} + \psi^{X,F}(1-\sigma)\text{Diag}(\boldsymbol{s^F})\right)\\
\boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{X}} &\equiv \psi^{X,F}\boldsymbol{I}\\
\boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{p}} &\equiv \psi^{X,F}(\sigma-1)(\boldsymbol{I}-\text{Diag}(\boldsymbol{s^F}))\boldsymbol{\Omega^H}\\
\boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{1+t^{av}}} &\equiv \boldsymbol{\Omega^F}
\end{aligned}
$$

Domestic Output Block. The second equation relates changes in spending, changes region-sector prices, and changes in Foreign price indices using Equations (C.16), (C.9) (C.15), (C.14), (E.8), (E.4) and (E.5). We have

$$
\begin{aligned}
Y_{rs} &= D^F_{rs} + D^H_{rs}\\
\text{where} \quad Y_{rs} &= L_{rs}(p_{rs})^{1/\alpha_s}\prod_{k\in\mathcal{S}}[\alpha_{ks}/P_{rk}]^{\alpha_{ks}/\alpha_s}\\
D^F_{rs} &= \delta_{rs}(p_{rs})^{1-1/\psi^{M,F}}\\
D^H_{rs} &= \sum_{d\in\mathcal{R}_H} X^H_{rds}\\
\delta_{rs} &= \sum_{i\in\mathcal{R}_F}(\theta_{ris})^{-(1-1/\psi^{M,F})}\sum_{h\in\mathcal{H}_s}\left(\theta^{M,F}_{rih}\right)^{1/\psi^{M,F}}
\end{aligned}
$$

$$
\begin{aligned}
X^H_{ors} &= \frac{\tilde{\theta}^H_{ors}[p_{os}]^{1-\sigma}}{[P^H_{rs}]^{1-\sigma}}X^H_{rs}\\
X^H_{rs} &= \frac{\theta^H_{rs}[P^H_{rs}]^{1-\sigma}}{[P_{rs}]^{1-\sigma}}X_{rs}\\
P_{rk} &= \left[\sum_{c=H,F}\theta^c_{rk}[P^c_{rk}]^{1-\sigma}\right]^{\frac{1}{1-\sigma}}\\
P^H_{rk} &= \left[\sum_{o\in\mathcal{R}_H}\tilde{\theta}^H_{ork}[p_{ok}]^{1-\sigma}\right]^{\frac{1}{1-\sigma}}
\end{aligned}
$$

Define the $|\mathcal{R}_F||\mathcal{S}|$-dimensional matrices and vectors

$$[\boldsymbol{A}]_{rs,dk} \equiv \frac{\alpha_{ks}}{\alpha_s}\mathbb{1}_{d=r} \qquad [\boldsymbol{\alpha}^{\boldsymbol{w}}]_{rs} \equiv \alpha_s$$

$$[\boldsymbol{\Omega}^{\boldsymbol{D}}]_{rs,dk} \equiv \frac{X^H_{rds}}{Y_{rs}}\mathbb{1}_{s=k} \qquad [\boldsymbol{\sigma}^{\boldsymbol{p}}]_{rs} \equiv \frac{D^F_{rs}}{Y_{rs}}(1-1/\psi^{M,F}) + \frac{D^H_{rs}}{Y_{rs}}(1-\sigma)$$

$$[\boldsymbol{\sigma}^{\boldsymbol{H}}]_{rs} \equiv -(1-\sigma)(1-\boldsymbol{s}^{\boldsymbol{F}}_{rs})$$

Log-linearizing these equations implies the following matrix equation:

$$\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{p}} d\log \boldsymbol{p} = \boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{X}} d\log \boldsymbol{X} + \boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{P}^{\boldsymbol{F}}} d\log \boldsymbol{P}^{\boldsymbol{F}} \tag{F.2}$$

$$\text{where} \quad \boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{p}} \equiv \left(\text{Diag}(\boldsymbol{\alpha}^{\boldsymbol{w}})^{-1} - \text{Diag}(\boldsymbol{\sigma}^{\boldsymbol{p}})\right) - \left(\boldsymbol{A}\cdot\text{Diag}(1-\boldsymbol{s}^{\boldsymbol{F}}) + \boldsymbol{\Omega}^{\boldsymbol{D}}\text{Diag}(\boldsymbol{\sigma}^{\boldsymbol{H}})\right)\boldsymbol{\Omega}^{\boldsymbol{H}}$$

$$\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{X}} \equiv \boldsymbol{\Omega}^{\boldsymbol{D}}$$

$$\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{P}^{\boldsymbol{F}}} \equiv ((\sigma-1)\boldsymbol{\Omega}^{\boldsymbol{D}} + \boldsymbol{A})\text{Diag}(\boldsymbol{s}^{\boldsymbol{F}})$$

Domestic Expenditures Block The third equation relates changes in spending, changes region-sector prices, changes in Foreign price indices, and changes in tariffs using Equations (C.13), (C.9) (E.1), (E.18), (E.4), (E.5), (E.11), and (E.15). We have

$$X_{rs} = \xi_{rs} + \gamma_{rs}N_r\tau + \sum_{k\in\mathcal{S}}(\gamma_{rs}\alpha_k + \alpha_{sk})Y_{rk}$$

$$\text{where} \quad Y_{rs} = L_{rs}(p_{rs})^{1/\alpha_s}\prod_{k\in\mathcal{S}}[\alpha_{ks}/P_{rk}]^{\alpha_{ks}/\alpha_s}$$

$$\tau = \frac{1}{N}\sum_{i\in\mathcal{R}_F}\sum_{r\in\mathcal{R}_H}\sum_{s\in\mathcal{S}}\sum_{h\in\mathcal{H}_s}\frac{t^{\text{av}}_{ih}}{1+t^{\text{av}}_{ih}}X^F_{irsh}$$

$$X^F_{irsh} = \frac{\varphi_{irsh}}{\sum_{o\in\mathcal{R}_F, v\in\mathcal{H}_s}\varphi_{orsv}}\frac{\theta^F_{rs}[P^F_{rs}]^{1-\sigma}}{[P_{rs}]^{1-\sigma}}X_{rs},$$

$$P_{rk} = \left[\sum_{c=H,F}\theta^c_{rk}[P^c_{rk}]^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

$$P^H_{rk} = \left[\sum_{o\in\mathcal{R}_H}\tilde{\theta}^H_{ork}[p_{ok}]^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

$$\varphi_{irsh} = \left[\theta^{X,F}_{irh}(1+t^{\text{av}}_{ih})\right]^{\frac{1-\sigma}{1+\psi^{X,F}\sigma}}(\theta^F_{irsh})^{\frac{1+\psi^{X,F}}{1+\psi^{X,F}\sigma}}$$

$$\mu_{rs} = \sum_{h\in\mathcal{H}_s}\sum_{i\in\mathcal{R}_F}\varphi_{irsh}$$

Define the $|\mathcal{R}_F||\mathcal{S}|$-dimensional matrices and vectors

$$[\boldsymbol{A^I}]_{rs,dk} \equiv \frac{Y_{rk}}{X_{rs}}(\gamma_{rs}\alpha_k + \alpha_{sk})'\mathbb{1}_{d=r} \qquad [\boldsymbol{s^\tau}]_{rs} \equiv \frac{\gamma_{rs}N_r}{X_{r,s}N}$$

$$[\boldsymbol{T^s}]_{rs} \equiv \sum_{i\in\mathcal{R}_F}\sum_{h\in\mathcal{H}_s} T_{irsh} \qquad [\boldsymbol{\omega^T}]_{oh} \equiv \sum_d \frac{X^F_{ods(h)h}}{1+t^{\text{av}}_{oh}} + \frac{1-\sigma}{1+\psi^{X,F}\sigma}\sum_{d\in\mathcal{R}_H}\left(T_{ods(h)h} - T_{rs(h)}\frac{X^F_{ods(h)v}}{X^F_{ds(h)}}\right)$$

Log-linearizing these equations implies the following matrix equation:

$$\begin{aligned}
\boldsymbol{\mathcal{E}^X_X} d\log \boldsymbol{X} &= \boldsymbol{\mathcal{E}^X_p} d\log \boldsymbol{p} + \boldsymbol{\mathcal{E}^X_{P^F}} d\log \boldsymbol{P^F} + \boldsymbol{\mathcal{E}^X_{1+t^{av}}} d\log(\boldsymbol{1}+\boldsymbol{\tilde{t}}) && \text{(F.3)}\\
\text{where}\quad \boldsymbol{\mathcal{E}^X_X} &\equiv \boldsymbol{I} - \boldsymbol{s^\tau}\boldsymbol{T^{s\prime}}\\
\boldsymbol{\mathcal{E}^X_p} &\equiv \boldsymbol{A^I}\text{Diag}(\boldsymbol{\alpha^w})^{-1} - \left(\boldsymbol{A^I}\boldsymbol{A} + (1-\sigma)\boldsymbol{s^\tau}\boldsymbol{T^{s\prime}}\right)(\boldsymbol{I} - \text{Diag}(\boldsymbol{s^F}))\boldsymbol{\Omega^H}\\
\boldsymbol{\mathcal{E}^X_{P^F}} &\equiv (1-\sigma)s^\tau \boldsymbol{T}^s - \left(\boldsymbol{A^I}\boldsymbol{A} + (1-\sigma)\boldsymbol{s^\tau}\boldsymbol{T^{s\prime}}\right)\text{Diag}(\boldsymbol{s^F})\\
\boldsymbol{\mathcal{E}^X_{1+t^{av}}} &\equiv \boldsymbol{s^\tau}\boldsymbol{\omega^{T\prime}}
\end{aligned}$$

Jacobians with Respect to Tariff Changes Substituting equations (F.1)-(F.3) into one another implies

$$d\log \boldsymbol{X} = \boldsymbol{A^X} d\log(\boldsymbol{1}+\boldsymbol{\tilde{t}}) \tag{F.4}$$

$$d\log \boldsymbol{P^F} = \boldsymbol{A^{P^F}} d\log(\boldsymbol{1}+\boldsymbol{\tilde{t}}) \tag{F.5}$$

$$d\log \boldsymbol{p} = \boldsymbol{A^p} d\log(\boldsymbol{1}+\boldsymbol{\tilde{t}}) \tag{F.6}$$

where

$$
\begin{aligned}
\boldsymbol{A}^{\boldsymbol{X}} &\equiv \left(\boldsymbol{Q}^{\boldsymbol{X}}_{\boldsymbol{X}} - \boldsymbol{Q}^{\boldsymbol{X}}_{\boldsymbol{P^F}}(\boldsymbol{Q}^{\boldsymbol{P^F}}_{\boldsymbol{P^F}})^{-1}\boldsymbol{Q}^{\boldsymbol{P^F}}_{\boldsymbol{X}}\right)^{-1}(\boldsymbol{\mathcal{E}}^{\boldsymbol{X}}_{\boldsymbol{1+t^{av}}} + \boldsymbol{Q}^{\boldsymbol{X}}_{\boldsymbol{P^F}}(\boldsymbol{Q}^{\boldsymbol{P^F}}_{\boldsymbol{P^F}})^{-1}\boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{1+t^{av}}})\\
\boldsymbol{A}^{\boldsymbol{P^F}} &\equiv \left(\boldsymbol{Q}^{\boldsymbol{P^F}}_{\boldsymbol{P^F}} - \boldsymbol{Q}^{\boldsymbol{P^F}}_{\boldsymbol{X}}(\boldsymbol{Q}^{\boldsymbol{X}}_{\boldsymbol{X}})^{-1}\boldsymbol{Q}^{\boldsymbol{X}}_{\boldsymbol{P^F}}\right)^{-1}(\boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{1+t^{av}}} + \boldsymbol{Q}^{\boldsymbol{P^F}}_{\boldsymbol{X}}(\boldsymbol{Q}^{\boldsymbol{X}}_{\boldsymbol{X}})^{-1}\boldsymbol{\mathcal{E}}^{\boldsymbol{X}}_{\boldsymbol{1+t^{av}}})\\
\boldsymbol{A}^{\boldsymbol{p}} &\equiv (\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{p}})^{-1}\left(\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{X}}\boldsymbol{A}^{\boldsymbol{X}} + \boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{P^F}}\boldsymbol{A}^{\boldsymbol{P^F}}\right)\\
\boldsymbol{Q}^{\boldsymbol{P^F}}_{\boldsymbol{P^F}} &\equiv \boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{P^F}} - \boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{p}}(\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{p}})^{-1}\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{P^F}}\\
\boldsymbol{Q}^{\boldsymbol{P^F}}_{\boldsymbol{X}} &\equiv \boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{X}} + \boldsymbol{\mathcal{E}}^{\boldsymbol{P^F}}_{\boldsymbol{p}}(\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{p}})^{-1}\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{X}}\\
\boldsymbol{Q}^{\boldsymbol{X}}_{\boldsymbol{X}} &\equiv \boldsymbol{\mathcal{E}}^{\boldsymbol{X}}_{\boldsymbol{X}} - \boldsymbol{\mathcal{E}}^{\boldsymbol{X}}_{\boldsymbol{p}}(\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{p}})^{-1}\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{X}}\\
\boldsymbol{Q}^{\boldsymbol{X}}_{\boldsymbol{P^F}} &\equiv \boldsymbol{\mathcal{E}}^{\boldsymbol{X}}_{\boldsymbol{P^F}} + \boldsymbol{\mathcal{E}}^{\boldsymbol{X}}_{\boldsymbol{p}}(\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{p}})^{-1}\boldsymbol{\mathcal{E}}^{\boldsymbol{p}}_{\boldsymbol{P^F}}
\end{aligned}
$$

Domestic Prices, Consumer Prices, Wages, Imports, and Terms of Trade. We now use the Jacobian matrices described in equations (F.4)-(F.6) to compute the Jacobian matrices of domestic price indices, consumer price indices, wages, imports and terms of trade with respect to tariffs.[5]

i. Domestic prices: From equations (E.4) and (E.5), we have

$$
P_{rk} = \left[\theta^F_{rk}[P^F_{rk}]^{1-\sigma} + \sum_{o\in\mathcal{R}_H}\theta^H_{rk}\tilde{\theta}^H_{ork}[p_{ok}]^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.
$$

Log-linearizing and substituting equations (F.5) and (F.6) implies

$$
\begin{aligned}
d\log \boldsymbol{P} &= \boldsymbol{A}^{\boldsymbol{P}} d\log(\boldsymbol{1}+\tilde{\boldsymbol{t}}) \qquad \text{(F.7)}\\
\text{where}\quad \boldsymbol{A}^{\boldsymbol{P}} &\equiv \left[\boldsymbol{I} - \text{Diag}(\boldsymbol{s}^{\boldsymbol{F}})\right]\boldsymbol{\Omega}^{\boldsymbol{H}}\boldsymbol{A}^{\boldsymbol{p}} + \text{Diag}(\boldsymbol{s}^{\boldsymbol{F}})\boldsymbol{A}^{\boldsymbol{P^F}}
\end{aligned}
$$

ii. Consumer prices: From equation (C.12), we have

$$
P^C_r = \prod_{k\in\mathcal{S}}(P_{rk})^{\gamma_k}.
$$

[5]We follow analogous steps to compute fiscal externalities through tariffs omitted from estimation, production subsidies, and income taxes. We omit these derivations for brevity.

Log-linearizing and substituting equation (F.7) implies

$$\begin{aligned} d\log \boldsymbol{P^C} &= \boldsymbol{A}^{\boldsymbol{P^C}} d\log(\boldsymbol{1}+\tilde{\boldsymbol{t}}) \\ \text{where} \quad [\boldsymbol{A}^{\boldsymbol{P^C}}]_{r,ih} &\equiv \sum_{s\in\mathcal{S}} \gamma_s [\boldsymbol{A^P}]_{rs,ih} \end{aligned}$$

iii. Wages: Since labor is the only factor and labor endowments are fixed, the change in wages w_{rs} is the same as the change in value added Y_{rs}. We have from equation (C.9) that

$$Y_{rs} = \phi_{rs} N_{rs} (p_{rs})^{1/\alpha_s} \prod_{k\in\mathcal{S}} [\alpha_{ks}/P_{rk}]^{\alpha_{ks}/\alpha_s}.$$

Log-linearizing and substituting equations (F.6) and (F.7) implies

$$\begin{aligned} d\log \boldsymbol{w} &= \boldsymbol{A^w} d\log(\boldsymbol{1}+\tilde{\boldsymbol{t}}) \\ \text{where} \quad \boldsymbol{A^w} &\equiv \text{Diag}(\boldsymbol{\alpha}^w)^{-1}\boldsymbol{A^p} - \boldsymbol{A}\boldsymbol{A^P} \end{aligned}$$

iv. Imports: From equations (15), (E.9), (E.18), (E.7), (E.4), and (E.5) we know

$$\begin{aligned} p^{X,F}_{irh} &= \theta^{X,F}_{irh} (q^{X,F}_{irh})^{\psi^{X,F}} \\ p_{irh} &= \left[\theta^{X,F}_{irh}(1+\tilde{t}^{\text{av}}_{ih})\right]^{\frac{1}{1+\psi^{X,F}}} (X^F_{irsh})^{\frac{\psi^{X,F}}{1+\psi^{X,F}}} \\ X^F_{irsh} &= \frac{\varphi_{irsh}}{\mu_{rs}} X^F_{rs} \\ X^F_{rs} &= \frac{\theta^F_{rs}[P^F_{rs}]^{1-\sigma}}{[P_{rs}]^{1-\sigma}} X_{rs} \\ P_{rk} &= \left[\theta^F_{rk}[P^F_{rk}]^{1-\sigma} + \sum_{o\in\mathcal{R}_H} \theta^H_{rk}\tilde{\theta}^H_{ork}[p_{ok}]^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \end{aligned}$$

We denote total imports of h from i by

$$m_{ih} \equiv \sum_{r\in\mathcal{R}_H} q^{X,F}_{irh}.$$

Log-linearizing, using the fact that $p_{irh} = p^{X,F}_{irh}(1+t^{av}_{ih})$, and using equations F.4, F.5, and F.7 implies

$$
\begin{aligned}
d \ln \boldsymbol{m} &= \boldsymbol{A}^{\boldsymbol{m}} d \ln(\mathbf{1}+\tilde{\boldsymbol{t}}) \\
\text{where} \quad \boldsymbol{A}^{\boldsymbol{m}} &\equiv \frac{1}{1+\psi^{X,F}}\left[\boldsymbol{\Psi}+\boldsymbol{\Omega}^{\boldsymbol{X}}\left(\boldsymbol{A}^{\boldsymbol{X}}+(1-\sigma)\left(\boldsymbol{A}^{\boldsymbol{P}^{\boldsymbol{F}}}-\boldsymbol{A}^{\boldsymbol{P}}\right)\right)\right] \\
[\boldsymbol{\Psi}]_{ih,ov} &= -\mathbb{1}_{i=o,h=v}\frac{(1+\psi^{X,F})\sigma}{1+\psi^{X,F}\sigma}-\frac{1-\sigma}{1+\psi^{X,F}\sigma}\mathbb{1}_{s(h)=s(v)}\sum_{r\in\mathcal{R}_H}\frac{q_{irh}^{X,F}}{m_{ih}}\frac{X_{ors(v)v}^{F}}{X_{rs(v)}^{F}} \\
[\boldsymbol{\Omega}^{\boldsymbol{X}}]_{ih,rs} &= \mathbb{1}_{h\in\mathcal{H}_s}\frac{q_{irh}^{X,F}}{m_{ih}}
\end{aligned}
$$

v. Terms of trade: The terms-of-trade effects that Home experiences from a change in export and import prices, respectively, vis-a-vis country-sector (i,s) are given by

$$
\begin{aligned}
d\mathrm{TOT}_{is}^{X,F} &= \sum_{r,h\in\mathcal{H}_s} q_{irh}^{X,F}\,\mathrm{d}p_{irh}^{X,F} \\
d\mathrm{TOT}_{is}^{M,F} &= -\sum_{r,h\in\mathcal{H}_s} q_{irh}^{M,F}\,\mathrm{d}p_{irh}^{M,F}
\end{aligned}
$$

Note that we normalize terms-of-trade effects such that a positive terms-of-trade effect corresponds to an aggregate welfare decrease in Home. From equations (15), (16), (E.18), (E.7), and (C.1) (and the absence of export taxes) we know

$$
\begin{aligned}
p_{irh}^{X,F} &= \theta_{irh}^{X,F}(q_{irh}^{X,F})^{\psi^{X,F}} \\
p_{rih}^{M,F} &= \theta_{rih}^{M,F}(q_{rih}^{M,F})^{-\psi^{M,F}} \\
X_{irsh}^{F} &= \frac{\varphi_{irsh}}{\mu_{rs}}X_{rs}^{F} \\
X_{rs}^{F} &= \frac{\theta_{rs}^{F}[P_{rs}^{F}]^{1-\sigma}}{[P_{rs}]^{1-\sigma}}X_{rs} \\
p_{irh}^{X,F} &= (\theta_{irh})^{-1}p_{rs(h)},
\end{aligned}
$$

Log-linearizing and using equations F.4, F.5, F.7, and F.6 implies

$$
\begin{aligned}
d\boldsymbol{ToT}^{\boldsymbol{X},\boldsymbol{F}} &= \boldsymbol{A}^{\boldsymbol{TOT}^{\boldsymbol{X},\boldsymbol{F}}} d\ln(\mathbf{1}+\tilde{\boldsymbol{t}}) \\
\text{where} \quad \boldsymbol{A}^{\boldsymbol{TOT}^{\boldsymbol{X},\boldsymbol{F}}} &\equiv \frac{\psi^{X,F}}{\psi^{X,F}+1}\left[\boldsymbol{\Gamma} - \tilde{\mathbf{X}}\left(\boldsymbol{A}^{\boldsymbol{X}} + (1-\sigma)(\boldsymbol{A}^{\boldsymbol{P}^{\boldsymbol{F}}} - \boldsymbol{A}^{\boldsymbol{P}})\right)\right] \\
[\boldsymbol{\Gamma}]_{is,oh} &\equiv \mathbb{1}_{i=o,s(h)=s}\frac{(1+\psi^{X,F})\sigma}{1+\psi^{X,F}\sigma}\sum_{r\in\mathcal{R}_H}\frac{X^F_{irsh}}{1+t^{av}_{ih}} \\
&\quad + \mathbb{1}_{s(h)=s}\frac{1-\sigma}{1+\psi^{X,F}\sigma}\sum_{r\in\mathcal{H}_s}\left(\sum_{v\in\mathcal{H}_s}\frac{X^F_{irsv}}{1+t^{av}_{iv}}\right)\frac{X^F_{orsh}}{X^F_{rs}} \\
[\tilde{\mathbf{X}}]_{is,rk} &\equiv -\mathbb{1}_{s=k}\sum_{r,h\in\mathcal{H}_s}\frac{X^F_{irsh}}{1+t^{av}_{ih}}
\end{aligned}
$$

and

$$
\begin{aligned}
d\boldsymbol{ToT}^{\boldsymbol{M},\boldsymbol{F}} &= \boldsymbol{A}^{\boldsymbol{ToT}^{\boldsymbol{M},\boldsymbol{F}}} d\ln\boldsymbol{p} \\
\text{where} \quad [\boldsymbol{A}^{\boldsymbol{ToT}^{\boldsymbol{M},\boldsymbol{F}}}]_{is,rk} &\equiv \mathbb{1}_{s=k}\sum_{h\in\mathcal{H}_s} p^{M,F}_{irh} q^{M,F}_{irh}
\end{aligned}
$$

From Tariff to Import Changes. The last step of our derivation is to convert the Jacobian matrices above—which are derivatives with respect to tariff changes—into the Jacobian matrices that enter our estimating equation—which are derivatives with respect to import changes. We do so by multiplying each original Jacobian matrix by the inverse of the Jacobian matrix of imports with respect to tariffs:

$$
\begin{aligned}
\frac{d\log\boldsymbol{P}^{\boldsymbol{C}}}{d\log\boldsymbol{m}} &= \boldsymbol{A}^{\boldsymbol{P}^{\boldsymbol{C}}}[\boldsymbol{A}^{\boldsymbol{m}}]^{-1}, \\
\frac{d\log\boldsymbol{w}}{d\log\boldsymbol{m}} &= \boldsymbol{A}^{\boldsymbol{w}}[\boldsymbol{A}^{\boldsymbol{m}}]^{-1}, \\
\frac{d\boldsymbol{ToT}^{\boldsymbol{X},\boldsymbol{F}}}{d\log\boldsymbol{m}} &= \boldsymbol{A}^{\boldsymbol{w}}[\boldsymbol{A}^{\boldsymbol{m}}]^{-1}, \\
\frac{d\boldsymbol{ToT}^{\boldsymbol{M},\boldsymbol{F}}}{d\log\boldsymbol{m}} &= \boldsymbol{A}^{\boldsymbol{w}}[\boldsymbol{A}^{\boldsymbol{m}}]^{-1}.
\end{aligned}
$$

F.2 Jacobian Matrices with Respect to Foreign Tariffs

We now turn to the analytical expression for the derivative of wages with respect to foreign tariffs, which we use for the model-testing exercise in Section 3.5. As already discussed in footnote 27, changes in foreign ad valorem tariffs, $d\log(1+t^{F,\text{av}}_{ih})$, are equivalent to changes in foreign import demand shifters, $d \log \theta_{rih}^{M,F} = -d \log(1 + t_{ih}^{F,\text{av}})$ for all $r \in \mathcal{R}_H$. We can therefore characterize Jacobian matrices with respect to foreign tariffs by characterizing Jacobian matrices with respect to changes in foreign import demand shifters that are uniform across domestic regions, as we do below. In such cases we denote by $d \log \theta_{ih}^{M,F}$ the common value across all r of $d \log \theta_{rih}^{M,F}$.

Revisiting the Domestic Output Block. We now revisit the three key blocks of the model studied in Appendix (F.1). Of these, the import pricing and domestic expenditures blocks are as before, except for that we may now ignore changes in Home tariffs. However, the domestic output block changes since it depends on Home exports. Importantly, we now have

$$d \log \delta_{rs} = \frac{1}{\psi^{M,F}} \sum_{i \in \mathcal{R}_F} \sum_{h \in \mathcal{H}_s} \frac{\delta_{irsh}}{\delta_{rs}} d \log \theta_{ih}^{M,F}$$
$$\text{where} \quad \delta_{irsh} \equiv (\theta_{ris})^{-(1-1/\psi^{M,F})} \left(\theta_{rih}^{M,F}\right)^{1/\psi^{M,F}}$$

By equations (16) and (C.1) and the absence of export taxes,

$$\begin{aligned} & p_{rih}^{M,F} q_{rih}^{M,F} = (p_{ris}^{M,F})^{1-1/\psi^{M,F}} (\theta_{rih}^{M,F})^{1/\psi^{M,F}} = \delta_{irsh} p_{rs} \\ \implies \quad & \delta_{irsh} / \delta_{rs} = p_{rih}^{M,F} q_{rih}^{M,F} / D_{rs}^{F} \end{aligned}$$

Combining these steps with our earlier analysis implies

$$\begin{aligned} \boldsymbol{\mathcal{E}}_{\boldsymbol{p}}^{\boldsymbol{p}} d \log \boldsymbol{p} = & \boldsymbol{\mathcal{E}}_{\boldsymbol{X}}^{\boldsymbol{p}} d \log \boldsymbol{X} + \boldsymbol{\mathcal{E}}_{\boldsymbol{P^F}}^{\boldsymbol{p}} d \log \boldsymbol{P^F} + \boldsymbol{\mathcal{E}}_{\boldsymbol{\theta^{M,F}}}^{\boldsymbol{p}} d \log \boldsymbol{\theta^{M,F}} \\ \text{where} \quad [\boldsymbol{\mathcal{E}}_{\boldsymbol{\theta^{M,F}}}^{\boldsymbol{p}}]_{rs,ih} \equiv & \frac{1}{\psi^{M,F}} \mathbb{1}_{s=s(h)} \frac{p_{rih}^{M,F} q_{rih}^{M,F}}{Y_{rs}} \end{aligned} \tag{F.8}$$

Changes in Value Added with Respect to Foreign Demand. We combine the three blocks of the model analogously to Appendix (F.1) in order to arrive at Jacobians $d \log \boldsymbol{p} / d \log \boldsymbol{\theta^{M,F}}$ and $d \log \boldsymbol{P^F} / d \log \boldsymbol{\theta^{M,F}}$ of Home output prices and import price indices with respect to foreign demand shocks. Finally, to compute the changes in region-sector value added (or equivalently wage bills) with respect to foreign demand shocks, we use that (a) the Cobb-Douglas nature of production implies log changes in wage bills equal log changes in output and (b) from equations (C.9), (E.4), and (E.5) we know

$$Y_{rs} = L_{rs}(p_{rs})^{1/\alpha_s} \prod_{k \in \mathcal{S}} [\alpha_{ks}/P_{rk}]^{\alpha_{ks}/\alpha_s}$$

$$P_{rk} = \left[\theta^F_{rk}[P^F_{rk}]^{1-\sigma} + \sum_{o \in \mathcal{R}_H} \theta^H_{rk}\tilde{\theta}^H_{ork}[p_{ok}]^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

Log-linearizing implies

$$\frac{d\log \boldsymbol{w}}{d\log \boldsymbol{\theta}^{\boldsymbol{M},\boldsymbol{F}}} = \boldsymbol{\mathcal{E}}^{\boldsymbol{w}}_{\boldsymbol{p}} \frac{d\log \boldsymbol{p}}{d\log \boldsymbol{\theta}^{\boldsymbol{M},\boldsymbol{F}}} + \boldsymbol{\mathcal{E}}^{\boldsymbol{w}}_{\boldsymbol{P}^{\boldsymbol{F}}} \frac{d\log \boldsymbol{P}^{\boldsymbol{F}}}{d\log \boldsymbol{\theta}^{\boldsymbol{M},\boldsymbol{F}}}.$$

$$\text{where} \quad \boldsymbol{\mathcal{E}}^{\boldsymbol{W}}_{\boldsymbol{p}} \equiv \text{Diag}\left(\boldsymbol{\alpha}^{w}\right)^{-1} - \boldsymbol{A}\left[\boldsymbol{I} - \text{Diag}\left(\boldsymbol{s}^{\boldsymbol{F}}\right)\right]\boldsymbol{\Omega}^{\boldsymbol{H}}$$

$$\boldsymbol{\mathcal{E}}^{\boldsymbol{w}}_{\boldsymbol{P}^{\boldsymbol{F}}} \equiv -\boldsymbol{A}\text{Diag}\left(\boldsymbol{s}^{\boldsymbol{F}}\right)$$